Functions and Roots

Sometimes it is more convenient when working with polynomials to give them a name to refer to them by. When we name a polynomial like this, we call it a function and write it as below:

$$f(x) = x^2 + 2x + 1$$

This is pronounced “f of x”. If we want to find what values the function takes at, say $x = 3$, we write:

$$f(3) = (3)^2 + 2(3) + 1 = 16$$

The values for which $f(x) = 0$ are called its roots.

In the above function, we can see that $f(-1) = 0$, so $x = -1$ is a root of the function.

If $x = a$ is a root of some function, then $(x - a)$ is a factor of that function.

Above, we see that since $f(-1) = 0$,
$(x - (-1)) = (x + 1)$ must divide $x^2 + 2x + 1$

Through long division or observation, we can see that:

$$f(x) = (x + 1)(x + 1)$$

So, we have factored $f(x)$ into a form where each factor has no degree higher than 2, this means $f(x)$ has been completely factored and we are able to identify the roots of the function.

We know that zero multiplied by anything equals zero. So, to find when $f(x)=0$, we'll look at each factor separately and find when each equals zero.

The first factor of $f(x)$ is $(x+1)$. We know that $x+1=0$ when $x=-1$, so our first root is -1. Our second factor is the same as the first, so our second root will also be -1. Since we've gone through all the factors, we know we've found all the roots.

Identify the roots in the following functions. For some of the larger polynomials, it may be worthwhile to "guess" a value for x (i.e. a **small** value), to see if $f(x)=0$ for that value. If $f(x)=0$ for $x=a$ you know that $(x-a)$ divides the function.

1. $f(x)=x^3+6x^2+12x+8$

2. $f(x)=x^3+x^2-4x-4$

3. $f(x)=x^3+x^2-10x+8$

4. $f(x)=x^3+4x^2+3x$

5. $f(x)=x^3-2x^2-7x-4$

6. $f(x)=x^3-13x-12$

7. $f(x)=x^4-6x^3+5x^2+12x$

8. $f(x) = -12x^2 - x^3 + x^4$

9. $f(x) = x^5 - 10x^3 + 9x$

10. $f(x) = x^3 - 3x^2 + 3x - 1$