Logarithms and their Properties

Logarithms are a different way of expressing exponential functions. We know that:

$$10^3 = 1000$$

In terms of logarithms, this translates into:

$$\log_{10} 1000 = 3$$

Notice that the equation has shifted focus. The solution to the logarithmic equation $\log_{10} 1000 = x$, is the solution to $10^x = 1000$. Logarithms help us get a handle on exponents and solve equations involving them.

In general,

$y = \log_a x$ if and only if $a^y = x$

Logarithms have several interesting properties:

1) $\log_a a = 1$

2) $\log_a a^x = x$

3) $\log_a (xy) = \log_a (x) + \log_a (y)$

4) $\log_a \left(\dfrac{x}{y} \right) = \log_a (x) - \log_a (y)$

5) $\log_a (x^y) = y \log_a (x)$

Rule 5 is a more general form of Rule 2.

These properties can be used to help solve equations involving logarithms and exponents.

Example:

$$3 + 4^{x+2} = 67$$

$$4^{x+2} = 64$$

$$\log_4 4^{x+2} = \log_4 64$$

$$(x+2)\log_4 4 = \log_4 4^3$$

$$x + 2 = 3$$

$$x = 1$$

In the third line, we chose to take the logarithms with base 4 because we're trying to solve for x which is the exponent of 4.

Now, using the properties of logarithms, try these questions on your own.

1. $\dfrac{2^x + 3}{5} = 7$

2. $\log_x 5 = 7$

3. $5^{x+4} - 25 = 100$

4. $8 + 2^{3x-6} = 40$

5. $\log_2 2^x = 17$

6. $7 + 3^{x-2} = 34$

7. $\dfrac{9^{4x+3}}{3} = 27$

8. $-2 + \dfrac{6^{x+3}}{2} = 16$

9. $\log_2 \left({}^{1}\!/\!{}_{2} \right) = x$

10. $3 + 4^x = 35$

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