Rational Root Theorem

The Rational Root Theorem states that:

> ***If* a polynomial has rational roots, then they will be of the form:**
>
> $$\frac{a}{b}$$
>
> **where a is a factor of the constant term of the polynomial and b is a factor of the coefficient of the highest degree term of the polynomial.**

A few things to note:

1) This **only** applies to rational roots. The polynomial may also have irrational roots.
2) The "if" at the beginning is important. This theorem only gives us a list of potential candidates for the rational roots. It sort of narrows down the playing field for us.
3) Once we have our candidates, if we check them all and none turn out to be *actual* roots, then we know the polynomial only has irrational roots.

Suppose we wanted to find the rational roots of:

$$f(x) = x^4 + 2x^2 - 9x + 5$$

This polynomial doesn't appear to factor and finding it's roots could otherwise take a while. But with the rational root theorem, we find that if it has a rational root, then the numerator must be a factor of 5 and the denominator must be a factor of the coefficient of the highest degree term (in this case, x^4 : the coefficient is 1).

So, our "candidates" for rational roots are:

$$\frac{5}{1}, -\frac{5}{1}, \frac{1}{1}, -\frac{1}{1}$$

Now, we check to see if any of these values actually **are** roots:

$$f(-5) = 725$$
$$f(5) = 635$$
$$f(1) = -1$$
$$f(-1) = 17$$

Since, a value is only a root if $f(x) = 0$, we see that the equation only has irrational roots.

Go through these questions and make a list of potential roots. Then, see which (if any) are actual roots. Don't worry about the irrational roots for now.

1. $f(x) = x^3 + 4x^2 - 9$

2. $f(x) = x^3 - x^2 + 6x - 6$

3. $f(x) = 3x^5 + 2x^3 - 2x^2 - 5$

4. $f(x) = x^4 - 5x^2 + 6$

5. $f(x) = x^2 - 2$

6. $f(x) = 4x^3 - x^2 - 12x + 3$

7. $f(x) = 2x^3 - 17x^2 + 27x - 9$

8. $f(x) = 45x^2 - 4x - 1$

9. $f(x) = x^4 - 6x^3 + 11x^2 - 6x$

10. $f(x) = 3x^3 + 11x^2 - 18x + 28$

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