

The Quadratic Formula

A *quadratic* equation is an equation where the highest degree term is x^2 . If we are trying to find the roots of a quadratic equation that doesn't factor so easily, we can use the quadratic formula:

A quadratic polynomial $ax^2 + bx + c$ has as its roots:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice the \pm sign. This means that the equation will give us **up to two** roots for a quadratic equation. Why two? Well, look at the equation $x^2 = 3$. The solutions to this equation are $x = \sqrt{3}$ and $x = -\sqrt{3}$. Similarly, any quadratic can have up to two roots. (Try and figure out on your own in what kinds of cases the quadratic formula would only give one root and other cases where it would give no real roots. Think about the +/- sign and the square root)

When you use the quadratic formula, it's best not to calculate the number on a calculator, as this will only give an approximation. Instead leave it as a fraction.

Follow through this example:

Find the roots of $4x^2 + x - 5$

In this equation, $a = 4$, $b = 1$ and $c = -5$. Now, we plug these values into the formula to get the roots.

The roots are:
$$\frac{-1 + \sqrt{1^2 - 4(4)(-5)}}{2(4)} = \frac{8}{8} = 1$$

$$\text{and } \frac{-1 - \sqrt{1^2 - 4(4)(-5)}}{2(4)} = \frac{-10}{8} = -\frac{5}{4}$$

Find the roots of these polynomials by using the quadratic formula:

1. $3x^2 + x - 4$

2. $x^2 - 5x + 9$

3. $2x^2 + 6x + 5$

4. $4x^2 + 2x + 2$

5. $-3x^2 + 5x - 8$

6. $-5x^2 + 3x - 3$

7. $x^2 - 4x + 2$

8. $9x^2 - 7x - 9$

9. $2x^2 + 3x + 2$

10. $x^2 + x + 1$