Simplifying Roots

A square root can be expressed in terms of as exponents as:

$$\sqrt{x} = x^{1/2}$$

Similarly, other roots can be expressed in the same way:

$$\sqrt[3]{x} = x^{1/3}$$
$$\sqrt[10]{x} = x^{1/10}$$

Using this new exponent way of writing a root, we can get a few interesting properties. (Make sure you understand properties of exponents before going through this worksheet!)

We know that:

$$(x^a)^b = x^{ab} = x^{ba} = (x^b)^a$$

From this, we can get that: $x^{2/3} = (x^2)^{1/3} = (x^{1/3})^2$

So, $\sqrt[3]{x^2} = \left(\sqrt[3]{x}\right)^2$

Basically, we can move the exponent of 2 inside or outside of the cubed root sign because, in reality, cube roots are exponents also.

Next, let's say we want to simplify $\sqrt{8}$ and write it in a more convenient way.

We notice that $8 = 4 \times 2$. And we know that $(a^x)(b^x) = (ab)^x$. So, we can write:

$$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$$

We've simplified the expression for $\sqrt{8}$ a great deal by writing it in terms of a smaller root. We chose to write 8 as 4 x 2 because we knew 4 was a perfect square. If we had a cubed root, we would try and write the number as a multiple of a perfect cube and another number.

Try following the same procedure to simplify these problems:

1. $\sqrt{18}$

2. $\sqrt{48}$

3. $\sqrt{27}$

4. $\sqrt{75}$

5. $\sqrt[4]{32}$

6. $\sqrt{500}$

7. $\sqrt{80}$

8. $\sqrt[4]{162}$

9. $\dfrac{\sqrt[3]{54}}{3}$

10. $$\frac{\sqrt[3]{54}}{\sqrt[3]{16}}$$