

Prime Factors

A prime number is a number that has only two factors: 1 and itself. Examples of primes are 2, 3, 5, 7, and 11. Long ago, the Greeks proved that there are an infinite number of primes.

Every number can be written as a product of prime numbers. It is often useful to do this so that some important properties about a single number, or group of numbers, can be found.

Finding the prime factors is a step by step process: first you find one prime factor, then you continue working your way down until you have written the original number as a product of prime numbers.

Example: Write 30 as a product of prime numbers

First, I would notice that 30 is an **even** number. So that means its divisible by 2, and 2 is a prime, so I would write:

$$30 = 2 \times 15$$

Next, I focus on 15. 15 is divisible by 5 since it ends in a 5, and 5 is prime. So, we have:

$$30 = 2 \times 5 \times 3$$

Since all of 2, 5 and 3 are prime, we're finished.

The shortcuts you can use are:

1. If you're trying to find the prime factors of n , only check the prime numbers up to \sqrt{n} . Why? Well, if a number has a factor greater than \sqrt{n} , then it will have a corresponding factor less than \sqrt{n} .
2. If a number is **even**, then it is divisible by 2
3. If the digits of a number add up to 3, then it is divisible by 3.
4. If a number ends in 5 or 0, then it is divisible by 5.
5. If the **alternating** digits have equal sums, then the number is divisible by 11.

Example: **136202**

One set of alternating digits has a sum of: $1 + 6 + 0 = 7$

The other set has a sum of: $3 + 2 + 2 = 7$

Since these sums are equal, the number is divisible by 11.

6. If the **alternating** digits have different sums, but the sums have a difference that is a multiple of 11, the number is divisible by 11:

Example: **91916**

$$9 + 9 + 6 = 24$$

$$1 + 1 = 2$$

$$24 - 2 = 22 = 2 \times 11$$

These rules can help when decomposing a number into its prime factors.

If you are trying to decompose a number into its prime factors and you find that it

has no prime factors less than (or equal to) \sqrt{n} , then the number is prime.

Write the following numbers as a product of prime factors.

1. 91
2. 415
3. 93
4. 332
5. 167
6. 76
7. 231
8. 223
9. 52
10. 770.