Greatest Common Divisor

The greatest common divisor (often abbreviated to gcd) between two numbers is the greatest number that is a factor of both numbers. For example, the greatest common divisor of 6 and 4 is 2.

To find the greatest common divisor of two numbers, first you need to decompose the number and write it as a product of its prime factors.

Then, we use a chart to determine which prime factors will be "carried over" to the greatest common divisor. An example can illustrate this better:

Find the greatest common divisor of 24 and 36.

$$24 = 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$

	# of 2's	# of 3's
24	3	1
36	2	2

We form a table with the original 2 numbers in the left hand column and the different prime factors in the top row. In each square we write the number of times that a certain prime factor occurred for each number. For example: 3 occurs once in the prime decomposition of 24, so we write a 1 in the square corresponding to the "24" row and "3's" column. Once we have completed the chart, we look at each column and highlight the square with the smallest number. In the "2's" column, we highlight 2 since 2 is smaller than 3. In the "3's" column, we highlight 1.

To find the greatest common divisor, we multiply each prime factor by itself as many times as the number highlighted in its column. We highlighted 2 in the "2's" column and 1 in the "3's" column, so the greatest common divisor is:

$$2 \times 2 \times 3 = 12$$

Alternatively, we can use another method for calculating the greatest common divisor. In this method, we need to write each number as a product of its prime factors. The difference here is that we need to group the 2's together and group the 3's together and use exponents. So, we would write:

$$24 = 2^3 3^1$$

$$36 = 2^2 3^2$$

Now, the idea here is to go through each exponent that is listed (whether it be for one number or both) and to choose each prime to the lowest of the two powers. So above, we see that 2 is listed in both prime decompositions, to the powers of 3 and 2. Since 2 is the lower of the two, we select 2 squared. Next, we look at 3. We have a choice between 3 to the power of 1 and 3 squared. Since 1 is lower than 2, we select 3 to the power of 1.

Our greatest common divisor is now:

$$2^2 3^1 = 12$$

You might notice that each of these methods is essentially the same method presented two different ways.

Try finding the greatest common divisor of these groups of numbers on your own:

1. 19 and 18
2. 36 and 48
3. 100 and 200
4. 4 and 9
5. 3, 36 and 51
6. 135 and 360
7. 18, 27 and 108
8. 436 and 545
9. 96 and 104
10. What is the greatest common divisor of **any** two prime numbers?