# Matrix Multiplication

Both addition and subtraction of matrices worked in a way you might expect. Simply line up values in the same position and find their sum or difference. Matrix multiplication isn't so simple.

Firstly, you can multiply different sized matrices (not all matrices can be multiplied though).

The second difference between multiplying regular numbers and multiplying matrices is that **order matters**. Multiply matrices A and B will usually give a different result than multiplying B and A. Often the two answers won't even be the same size!

Here is a quick way to tell if two matrices can be multiplied. Write out the dimensions of the first matrix beside the dimensions of the second matrix. If the matrices can be multiplied, the two **inner** values will be the same. If the two inner values are the same, then the two outer values will give the dimensions of the matrix you will get when you multiply the two.

Let's say you wanted to multiply a 4 x 6 matrix with a 6 x 3 matrix. (Remember, when giving the dimensions of a matrix, give the number of rows first and then the number of columns)

So, we write: 4 x 6 6 x 3

The two inner dimensions are both 6, so we know we **can** multiply the matrices *in this order*. The outer numbers tell us that the answer will be a 4 x 3 matrix.

What if we wanted to multiply the matrices in the other order?

We'd write: 6 x 3 4 x 6

The inner numbers are 4 and 3. They don't match, so we'd know we can't multiply the matrices in this order.

Now, on to actual multiplication!

To multiply two matrices, it's best to start out by figuring out what size our answer will be.

Let's say I want to multiply a 2 x 3 matrix by a 3 x 2 matrix. We know our answer will be a 2 x 2 matrix (from the method above)

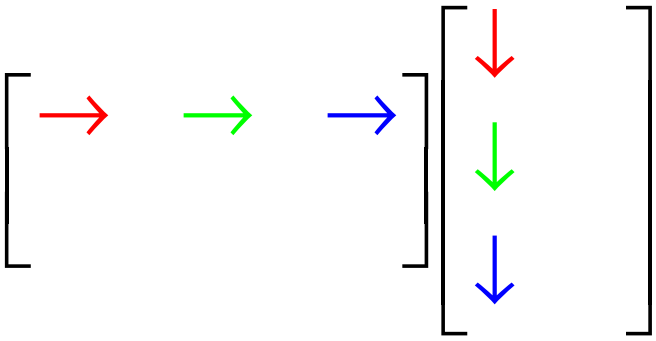
So, we start by writing a blank 2 x 2 matrix:

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

I start by choosing an entry I want to find. I'll start by finding the (1,1) entry.

This means the entry in row 1, column 1. So I highlight the first row in the first matrix and the first column in the second matrix.

I multiply the entries with same coloured arrows together and then sum the three numbers



If our matrices were:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 3 \end{bmatrix}$$

Our (1,1) entry in the answer would be:

$$(1)(2)+(2)(1)+(1)(3)=7$$

For the (1,2) entry we go across the 1st row of the first matrix and down the second column of the second matrix to get:

$$(1)(1)+(2)(2)+(1)(3)=8$$

The (2,1) entry is:

$$(2)(2)+(2)(1)+(1)(3)=9$$

And the (2,2) entry is:

$$(2)(1)+(2)(2)+(1)(3)=9$$

And our matrix looks like this:

$$\begin{bmatrix} 7 & 8 \\ 9 & 9 \end{bmatrix}$$

Now, try these on your own:

1. $$\begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \end{bmatrix}=$$

2. $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}=$$

3. $$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=$$

4. $$\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}1 & -2 & -1\\ 0 & 2 & 4\end{bmatrix}=$$

5. Which of these matrices can be multiplied together and which cannot be?

   a. A 5 x 4 matrix and a 3 x 2 matrix

   b. A 2 x 3 matrix and a 3 x 8 matrix

   c. A 4 x 2 matrix and a 2 x 3 matrix

   d. A 8 x 19 matrix and a 18 x 9 matrix

   e. A 2 x 1 matrix and a 4 x 2 matrix

   f. A 1 x 1 matrix and a 1 x 1 matrix

6. $$\begin{bmatrix}1 & 4 & -2\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}-1 & -1\\ 2 & 1\\ 3 & 1\end{bmatrix}=$$

7. $$\begin{bmatrix}-1 & -1\\ 2 & 1\\ 3 & 1\end{bmatrix}\begin{bmatrix}1 & 4 & -2\\ 0 & 1 & 1\end{bmatrix}=$$

8. $$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}^3=$$

9. $\begin{bmatrix} 1 & 2 & 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & 1 \\ 0 & 2 \\ 6 & 2 \\ 3 & 3 \end{bmatrix} =$

10. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^2 =$

http://math.about.com