Mathematical Induction

Mathematical induction is a way of proving a statement true for all positive integers. The way it works is that we prove the statement true for some small value (usually for 1 or 0). Then, we show that if the statement is true for $n = k$, then the statement **must** be true for $n = k + 1$. In this way, the proof is sort of like a domino effect. We've shown the statement true for n=1. But then, if it's true for $n = 1$, it must be true for $n = 1 + 1 = 2$. But, if the statement is true for $n = 2$, it must be true for $n = 2 + 1 = 3$, etc.

Example:

Show that $n(n+1)$ is always even (i.e. always a multiple of 2)

Step 1. Show this is true for a small value of n

When $n = 0$, we have $0(0+1) = 0$ which is even. So the statement is true for $n = 0$

Step 2. Assume that $n(n+1)$ is even for $n = k$ and show that this implies $n(n+1)$ is even for $n = k + 1$

We know $k(k+1)$ is even. From here, we have to try and end up with the statement that $(k+1)[(k+1)+1]$ is even

But,

$$(k+1)[(k+1)+1] = (k+1)(k+2)$$

$$= k(k+1) + 2(k+1)$$

The first term is even by our assumption and the second term is even because it is a multiple of 2. Thus, the statement is true for $n = k + 1$

Step 3. Since we know the statement is true for $n = 0$, and we know that if the statement is true for $n = k$, then it must be true for $n = k + 1$, then for all integers greater than 0, $n(n+1)$ is even.

Try proving the following statements by induction on your own. Some of them can be solved using other methods, but make sure you use induction to solve them.

1. $$1 + 2 + 3 + ... + (n-1) + n = \frac{n(n+1)}{2}$$

2. $$1^2 + 2^2 + 3^2 + ... + (n-1)^2 + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. The product of three consecutive numbers is always a multiple of 3

4. $$2 + 4 + 6 + ... + 2(n-1) + 2n = n^2 + n$$

5. $$1 + 3 + 5 + 7 + ... + (2n-1) = n^2$$

6. $$1 + 4 + 7 + 10 + ... + (3n-2) = \frac{n}{2}(3n-1)$$

7. $2^n \geq 2n$

8. $1+3+9+27+...+3^{n-1} = \frac{3^n - 1}{2}$

9. $2^n \geq 100n$ for $n \geq 10$

http://math.about.com